A Stochastic Bi-objective Location Model for Reverse Logistics

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XVII EWGLA Meeting, Elche
September 19th, 2008

Motivation

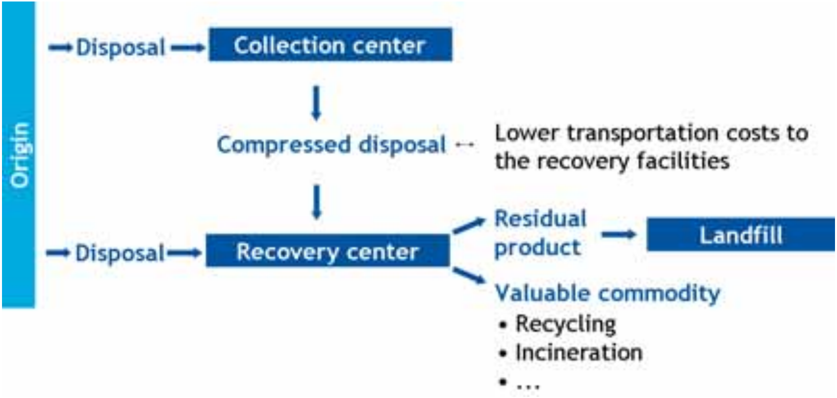
- Collecting, recycling, disposal is increasingly important
  - Environment aspects
  - Economic aspects
- Management Integrated Systems
- Problem studied in this work
  - Municipal
  - Regional
  - National

General Outline

A set of origins /populations ⇒ Different types of disposal are generated in each of them

- Paper
- Plastic
- Urban Solid Waste (USW)
- Glass
- ...

General Outline



Technologies

Different types of technology available for each type of disposal

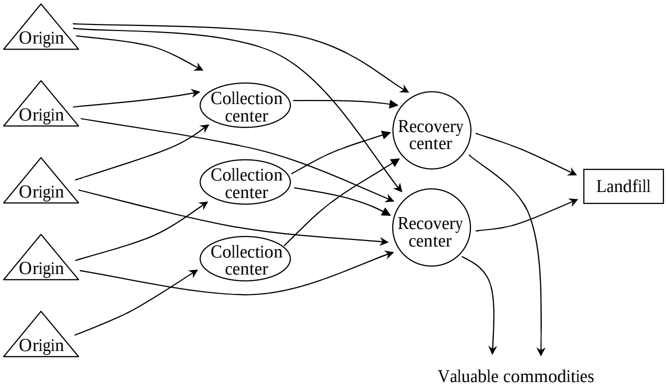
- Manual
- Automatic
- Semi-automatic

Different rates of conversion, costs, etc.

Assumptions

- We have a set of potential locations for installing
  - Collection centers
  - Recovery centers
  - Landfills
- Collection centers
  - Can be set-up for receiving any kind of disposal
  - There is a maximum operating capacity for each kind of disposal
- Recovery centers
  - Can receive any kind of disposal if the specific technology is installed
  - The technology is modular
  - There is a maximum number of modules of each type of technology for each type of disposal

Network Structure



Additional Assumptions

- Minimum levels are imposed on the recycling levels of some products (glass and paper)
  e.g. In most cases it is more profitable incinerating than recycling
- For a valuable commodity, there might be a maximum amount that can be sold
  e.g. Often there are limits on the amount that can be sent to the incinerators
- Each facility operates with a single technology for each product

Costs and Profits

- Costs
  - Set-up costs for installing the facilities (collection/recovery centers and landfills
  - Set-up cost for preparing a collection center to process each specific disposal
  - Set-up cost for installing a specific technology in each recovery center
  - Transportation costs
  - Disposal tax associated with the residual product
- Benefits
  - Profit associated with valuable commoditites

Environmental Impact

Obnoxious effect associated with the facilities

- Noise
- Odour
- ...

Function of the euclidian distances

We assume that incentives can be offered to the populations in order to decrease the negative feelings towards the facilities

Stochasticity

- Sources of stochasticity
  - Amount of each type of disposal generated
  - Transportation costs

- Embedding stochasticity
  - A set of scenarios
  - Probabilities associated with scenarios

In all...

Location Model

- Multiple echelons
- Multi commodity
- Stochastic
- Bi-objective
- Choice of technology

Two-stage Stochastic Mixed Integer Programming Problem

- 1st stage: strategic decisions
- 2nd stage: operational/tactical decisions

Index Sets

$\mathcal{P}_W$	:	set of disposal types
$\mathcal{P}_F$	:	set of valuable commodities
$\mathcal{G}$	:	set of different technologies available for the recovery centers
$\mathcal{J}$	:	set of origins of the disposals
$\mathcal{I}_C$	:	set of potential locations for the collection centers
$\mathcal{I}_R$	:	set of potential locations for recovery centers
$\mathcal{I}_L$	:	set of potential locations for the landfills
$\mathcal{S}$	:	set of scenarios

Parameters

O_{jps} : amount of disposal $p \in \mathcal{P}_W$ originated at origin $j \in \mathcal{J}$

KC_p : capacity of a collection center for processing product $p \in \mathcal{P}_W$

Q_{pg} : capacity of a module of technology $g \in \mathcal{G}$ when processing product $p \in \mathcal{P}_W$

KR_{ipg} : maximum number of modules of technology $g \in \mathcal{G}$ that can be installed at recovery center $i \in \mathcal{I}_R$ to process product $p \in \mathcal{P}_W$

A_{pqg} : maximum proportion of valuable commodity $p \in \mathcal{P}_F$ that can be obtained from product $q \in \mathcal{P}_W$ when using technology $g \in \mathcal{G}$

Parameters II

D^{O}_{ij} : road distance between origin $j \in \mathcal{J}$ and facility $i \in \mathcal{I}_C \cup \mathcal{I}_R$

$D^{NO}_{ii'}$: road distance between facility $i \in \mathcal{I}_R$ and facility $i' \in \mathcal{I}_C \cup \mathcal{I}_L$

MIN_{pq} : the minimum rate imposed for the conversion of product $q \in \mathcal{P}_W$ into product $p \in \mathcal{P}_F$

MAX_p : the maximum amount of valuable commodity $p \in \mathcal{P}_F$ that can be sold

Parameters III

OE^C_{ip} : the obnoxious effect due to the collection of product $p \in \mathcal{P}_W$ in collection center $i \in \mathcal{I}_C$

OE^R_{ip} : the obnoxious effect due to the processing of product $p \in \mathcal{P}_W$ in recovery center $i \in \mathcal{I}_R$

OE^L_i : the obnoxious effect due to the residual product in landfill $i \in \mathcal{I}_L$

p_s : probability of scenario $s \in \mathcal{S}$

Costs/Profits

FC_i : fixed cost for installing a collection center at $i \in \mathcal{I}_C$

FR_i : fixed cost for installing a recovery center at $i \in \mathcal{I}_R$

FL_i : fixed cost for installing a landfill at $i \in \mathcal{I}_L$

FCP_p : fixed cost for preparing a collection center for receiving disposal $p \in \mathcal{P}_W$

$FRPG_{ipg}$: fixed cost for installing a module of technology $g \in \mathcal{G}$ in location $i \in \mathcal{I}_R$ for processing disposal $p \in \mathcal{P}_W$

Costs/Profits II

CTO_{ps} : cost (euro per unit and per km) for shipping one unit of product $p \in \mathcal{P}_W$ from an origin to a collection center or to a recovery center under scenario $s \in \mathcal{S}$

CTC_{ps} : cost (euro per unit and per km) for shipping one unit of product $p \in \mathcal{P}_W$ from a collection center to a recovery center under scenario $s \in \mathcal{S}$

CTR_s : cost (euro per unit and per km) for shipping one unit of residual product from a recovery center to a landfill under scenario $s \in \mathcal{S}$

B_p : profit of each unit of valuable commodity $p \in \mathcal{P}_F$

DT : disposal tax per unit of residual product

(Strategic) Decision Variables

$$y_i^C = \begin{cases} 1 & \text{if collection center installed at } i \in \mathcal{I}_C \text{ is operating} \\ 0 & \text{otherwise} \end{cases}$$

$$y_i^R = \begin{cases} 1 & \text{if recovery center installed at } i \in \mathcal{I}_R \text{ is operating} \\ 0 & \text{otherwise} \end{cases}$$

$$y_i^L = \begin{cases} 1 & \text{if a landfill installed at } i \in \mathcal{I}_L \text{ is open} \\ 0 & \text{otherwise} \end{cases}$$

(Strategic) Decision Variables II

$$w_{ip}^{C} = \begin{cases} 1 & \text{if collection center } i \in \mathcal{I}_C \text{ is processing disposal } p \in \mathcal{P}_W \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ipg}^{R} = \begin{cases} 1 & \text{if recovery center } i \in \mathcal{I}_R \text{ is processing disposal } p \in \mathcal{P}_W \\ & \text{using technology } g \in \mathcal{G} \\ 0 & \text{otherwise} \end{cases}$$

n_{ipg}^{R} = number of modules of technology $g \in \mathcal{G}$ installed in recovery center $i \in \mathcal{I}_R$ to disposal $p \in \mathcal{P}_W$

(Tactical/Operational) Decision Variables

x^{OC}_{ijps} = amount of disposal $p \in \mathcal{P}_W$ sent from origin $j \in \mathcal{J}$ to collection center $i \in \mathcal{I}_C$ under scenario $s \in S$

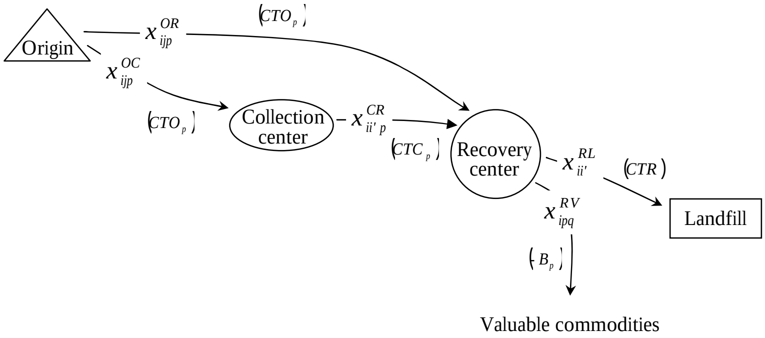
x^{OR}_{ijps} = amount of disposal $p \in \mathcal{P}_W$ sent from origin $j \in \mathcal{J}$ to recovery center $i \in \mathcal{I}_R$ under scenario $s \in S$

$x^{CR}_{ii'ps}$ = amount of disposal $p \in \mathcal{P}_W$ sent from collection center $i \in \mathcal{I}_C$ to recovery center $i' \in \mathcal{I}_R$ under scenario $s \in S$

$x^{RL}_{ii's}$ = amount of residual disposal obtained in recovery center $i \in \mathcal{I}_R$ sent to landfill $i' \in \mathcal{I}_L$ under scenario $s \in S$

x^{RV}_{ipqgs} = amount of valuable commodity $p \in \mathcal{P}_F$ obtained in recovery center $i \in \mathcal{I}_R$ from product $q \in \mathcal{P}_W$ using technology $g \in \mathcal{G}$ under scenario $s \in S$

Flow Variables and Unitary Transportation Costs



Objective Function 1. Costs

$$
\begin{aligned}
\text{MIN} \sum_{i \in \mathcal{I}_C} FC_i y_i^C + \sum_{i \in \mathcal{I}_R} FR_i y_i^R + \sum_{i \in \mathcal{I}_L} FL_i y_i^L \\
+ \sum_{p \in \mathcal{P}_W} \left(FCP_p \sum_{i \in \mathcal{I}_C} w_{ip}^C \right) + \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} \sum_{g \in \mathcal{G}} FRPG_{ipg} n_{ipg}^R \\
+ \sum_{s \in \mathcal{S}} \rho_s \Bigg[\sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}_C} \sum_{p \in \mathcal{P}_W} CTO_p D_{ij}^O x_{ijps}^{OC} + \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} CTO_p D_{ij}^O x_{ijps}^{OR} \\
+ \sum_{i \in \mathcal{I}_C} \sum_{i' \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} CTC_p D_{ii'}^{NO} x_{ii'ps}^{CR} + \sum_{i \in \mathcal{I}_R} \sum_{i' \in \mathcal{I}_L} CTR \times D_{ii'}^{NO} x_{ii's}^{RL} \\
+ DT \sum_{i \in \mathcal{I}_R} \sum_{i' \in \mathcal{I}_L} x_{ii's}^{RL} - \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_F} \sum_{q \in \mathcal{P}_W} B_p \sum_{g \in \mathcal{G}} x_{ipqgs}^{RV} \Bigg]
\end{aligned}
$$

Objective Function 1. Costs

$$
\begin{aligned}
\text{MIN} & \boxed{\sum_{i\in\mathcal{I}_C} FC_i y_i^C + \sum_{i\in\mathcal{I}_R} FR_i y_i^R + \sum_{i\in\mathcal{I}_L} FL_i y_i^L} \\
& + \sum_{p\in\mathcal{P}_W}\left(FCP_p \sum_{i\in\mathcal{I}_C} w_{ip}^C\right) + \sum_{i\in\mathcal{I}_R}\sum_{p\in\mathcal{P}_W}\sum_{g\in\mathcal{G}} FRPG_{ipg}\, n_{ipg}^R \\
& + \sum_{s\in\mathcal{S}} \rho_s \Bigg[\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}_C}\sum_{p\in\mathcal{P}_W} CTO_p D_{ij}^O x_{ijps}^{OC} + \sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}_R}\sum_{p\in\mathcal{P}_W} CTO_p D_{ij}^O x_{ijps}^{OR} \\
& + \sum_{i\in\mathcal{I}_C}\sum_{i'\in\mathcal{I}_R}\sum_{p\in\mathcal{P}_W} CTC_p D_{ii'}^{NO} x_{ii'ps}^{CR} + \sum_{i\in\mathcal{I}_R}\sum_{i'\in\mathcal{I}_L} CTR \times D_{ii'}^{NO} x_{ii's}^{RL} \\
& + DT \sum_{i\in\mathcal{I}_R}\sum_{i'\in\mathcal{I}_L} x_{ii's}^{RL} - \sum_{i\in\mathcal{I}_R}\sum_{p\in\mathcal{P}_F}\sum_{q\in\mathcal{P}_W} B_p \sum_{g\in\mathcal{G}} x_{ipqgs}^{RV}\Bigg]
\end{aligned}
$$

Objective Function 1. Costs

$$
\begin{aligned}
\text{MIN} & \sum_{i\in\mathcal{I}_C} FC_i y_i^C + \sum_{i\in\mathcal{I}_R} FR_i y_i^R + \sum_{i\in\mathcal{I}_L} FL_i y_i^L \\
& + \boxed{\sum_{p\in\mathcal{P}_W}\left(FCP_p \sum_{i\in\mathcal{I}_C} w_{ip}^C\right) + \sum_{i\in\mathcal{I}_R}\sum_{p\in\mathcal{P}_W}\sum_{g\in\mathcal{G}} FRPG_{ipg}\, n_{ipg}^R} \\
& + \sum_{s\in\mathcal{S}}\rho_s \Bigg[\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}_C}\sum_{p\in\mathcal{P}_W} CTO_p D_{ij}^O x_{ijps}^{OC} + \sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}_R}\sum_{p\in\mathcal{P}_W} CTO_p D_{ij}^O x_{ijps}^{OR} \\
& + \sum_{i\in\mathcal{I}_C}\sum_{i'\in\mathcal{I}_R}\sum_{p\in\mathcal{P}_W} CTC_p D_{ii'}^{NO} x_{ii'ps}^{CR} + \sum_{i\in\mathcal{I}_R}\sum_{i'\in\mathcal{I}_L} CTR \times D_{ii'}^{NO} x_{ii's}^{RL} \\
& + DT \sum_{i\in\mathcal{I}_R}\sum_{i'\in\mathcal{I}_L} x_{ii's}^{RL} - \sum_{i\in\mathcal{I}_R}\sum_{p\in\mathcal{P}_F}\sum_{q\in\mathcal{P}_W} B_p \sum_{g\in\mathcal{G}} x_{ipqgs}^{RV}\Bigg]
\end{aligned}
$$

Objective Function 1. Costs

$$
\begin{aligned}
\text{MIN} & \sum_{i\in\mathcal{I}_C} FC_i y_i^C + \sum_{i\in\mathcal{I}_R} FR_i y_i^R + \sum_{i\in\mathcal{I}_L} FL_i y_i^L \\
& + \sum_{p\in\mathcal{P}_W}\left(FCP_p \sum_{i\in\mathcal{I}_C} w_{ip}^C\right) + \sum_{i\in\mathcal{I}_R}\sum_{p\in\mathcal{P}_W}\sum_{g\in\mathcal{G}} FRPG_{ipg}\, n_{ipg}^R \\
& + \sum_{s\in\mathcal{S}}\rho_s \Bigg[\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}_C}\sum_{p\in\mathcal{P}_W} CTO_p D_{ij}^O x_{ijps}^{OC} + \sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}_R}\sum_{p\in\mathcal{P}_W} CTO_p D_{ij}^O x_{ijps}^{OR} \\
& + \sum_{i\in\mathcal{I}_C}\sum_{i'\in\mathcal{I}_R}\sum_{p\in\mathcal{P}_W} CTC_p D_{ii'}^{NO} x_{ii'ps}^{CR} + \sum_{i\in\mathcal{I}_R}\sum_{i'\in\mathcal{I}_L} CTR \times D_{ii'}^{NO} x_{ii's}^{RL} \\
& + DT \sum_{i\in\mathcal{I}_R}\sum_{i'\in\mathcal{I}_L} x_{ii's}^{RL} - \sum_{i\in\mathcal{I}_R}\sum_{p\in\mathcal{P}_F}\sum_{q\in\mathcal{P}_W} B_p \sum_{g\in\mathcal{G}} x_{ipqgs}^{RV}\Bigg]
\end{aligned}
$$

Objective Function 2. Obnoxious Effect

$$\text{MIN} \sum_{i \in \mathcal{I}_C} \sum_{p \in \mathcal{P}_W} OE^C_{ip} w^C_{ip} + \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} OE^R_{ip} \sum_{g \in \mathcal{G}} z^R_{ipg} + \sum_{i \in \mathcal{I}_L} OE^L_i y^L_i$$

Objective Function 2. Obnoxious Effect

$$\text{MIN} \boxed{\sum_{i \in \mathcal{I}_C} \sum_{p \in \mathcal{P}_W} OE_{ip}^C w_{ip}^C} + \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} OE_{ip}^R \sum_{g \in \mathcal{G}} z_{ipg}^R + \sum_{i \in \mathcal{I}_L} OE_i^L y_i^L$$

Objective Function 2. Obnoxious Effect

$$\text{MIN} \sum_{i \in \mathcal{I}_C} \sum_{p \in \mathcal{P}_W} OE^C_{ip} w^C_{ip} + \boxed{\sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} OE^R_{ip} \sum_{g \in \mathcal{G}} z^R_{ipg}} + \sum_{i \in \mathcal{I}_L} OE^L_i y^L_i$$

Objective Function 2. Obnoxious Effect

$$\text{MIN} \sum_{i \in \mathcal{I}_C} \sum_{p \in \mathcal{P}_W} OE^C_{ip} w^C_{ip} + \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} OE^R_{ip} \sum_{g \in \mathcal{G}} z^R_{ipg} + \boxed{\sum_{i \in \mathcal{I}_L} OE^L_i y^L_i}$$

Constraints I

All waste generated must be sent to a facility

$$\sum_{i \in \mathcal{I}_C} x_{ijps}^{OC} + \sum_{i \in \mathcal{I}_R} x_{ijps}^{OR} = O_{jps} \qquad p \in \mathcal{P}_W, j \in \mathcal{J}, s \in \mathcal{S}$$

Flow conservation in collection centers

$$\sum_{j \in \mathcal{J}} x_{ijps}^{OC} = \sum_{i' \in \mathcal{I}_R} x_{ii'ps}^{CR} \qquad i \in \mathcal{I}_C, p \in \mathcal{P}_W, s \in \mathcal{S}$$

Balance of valuable products obtained from waste products

$$A_{pqg} \left(\sum_{j \in \mathcal{J}} x_{ijqs}^{OR} + \sum_{i' \in \mathcal{I}_C} x_{i'iqs}^{CR} \right) \geq x_{ipqgs}^{RV} \qquad i \in \mathcal{I}_R, q \in \mathcal{P}_W, p \in \mathcal{P}_F, g \in \mathcal{G}$$

Constraints II

Residual product to be sent to landfills

$$\sum_{i' \in \mathcal{I}_L} x^{RL}_{ii's} = \sum_{q \in \mathcal{P}_W} \left(\sum_{j \in \mathcal{J}} x^{OR}_{ijqs} + \sum_{i' \in \mathcal{I}_C} x^{CR}_{i'iqs} \right) - \sum_{q \in \mathcal{P}_W} \sum_{p \in \mathcal{P}_F} \sum_{g \in \mathcal{G}} x^{RV}_{ipqgs} \qquad i \in \mathcal{I}_R,\ s$$

Capacity constraints for each technology at each recovery centers

$$\sum_{j \in \mathcal{J}} x^{OR}_{ijps} + \sum_{i' \in \mathcal{I}_C} x^{CR}_{i'ips} \leq \sum_{g \in \mathcal{G}} Q_{pg}\, n^{R}_{ipg} \qquad i \in \mathcal{I}_R,\ p \in \mathcal{P}_W,\ s \in \mathcal{S}$$

Constraints III

Minimum rates of conversion (law)

$$\sum_{i \in \mathcal{I}_R} \sum_{g \in \mathcal{G}} x^{RV}_{ipqgs} \geq MIN_{pq} \left(\sum_{j \in \mathcal{J}} O_{jqs} \right) \qquad q \in \mathcal{P}_W,\, p \in \mathcal{P}_F,\, s \in \mathcal{S}$$

Market constraints

$$\sum_{i \in \mathcal{I}_R} \sum_{q \in \mathcal{P}_W} \sum_{g \in \mathcal{G}} x^{RV}_{ipqgs} \leq MAX_p \qquad p \in \mathcal{P}_F,\, s \in \mathcal{S}$$

Capacity constraints for the collection centers

$$\sum_{j \in \mathcal{J}} x^{OC}_{ijps} \leq KC_p \omega^C_{ip} \qquad i \in \mathcal{I}_C,\, p \in \mathcal{P}_W,\, s \in \mathcal{S}$$

Constraints IV

Consistency for product conversion and the available technology

$$x^{RV}_{ipqgs} \leq \left(\sum_{j \in \mathcal{J}} O_{jq} \right) z^{R}_{ipg} \qquad i \in \mathcal{I}_R,\ q \in \mathcal{P}_W,\ p \in \mathcal{P}_F,\ g \in \mathcal{G},\ s \in \mathcal{S}$$

A collection center can only be used if it was installed

$$\omega^{C}_{ip} \leq y^{C}_{i} \qquad i \in \mathcal{I}_C,\ p \in \mathcal{P}_W$$

A landfill can only be used if it was installed

$$x^{RL}_{ii'} \leq \left(\sum_{j \in \mathcal{J}} \sum_{p \in \mathcal{P}_W} O_{jp} \right) y^{L}_{i} \qquad i \in \mathcal{I}_R,\ i' \in \mathcal{I}_L$$

Constraints V

Unicity of technology

$$\sum_{g \in \mathcal{G}} z_{ipg}^{R} \leq 1 \qquad i \in \mathcal{I}_R,\ p \in \mathcal{P}_W$$

Consistency in recovery centers (technology)

$$\sum_{g \in \mathcal{G}} z_{ipg}^{R} \leq y_i^{R} \qquad i \in \mathcal{I}_R,\ p \in \mathcal{P}_W$$

Constraint for the number of modules of each technology

$$n_{ipg}^{R} \leq KR_{ipg} z_{ipg}^{R} \qquad i \in \mathcal{I}_R,\ p \in \mathcal{P}_W,\ g \in \mathcal{G}$$

Province of Córdoba

- 78 towns
- 8 eligible locations: Collection, recovery and landfills
- 4 waste products: Glass, paper, USW, yellow
- 5 valuable products: Glass, paper, plastic, compost, incineration
- 2 technologies: Manual/automatic
- 9 scenarios

- 10144 constraints
- 48571 variables (64 integer, 120 binary)

Goals and Methodology

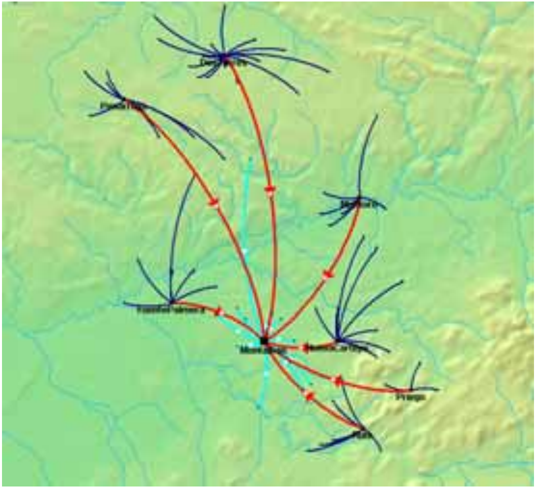
Goals:

- To obtain non-dominated solutions
- To check if these solutions can be found in reasonable computational times

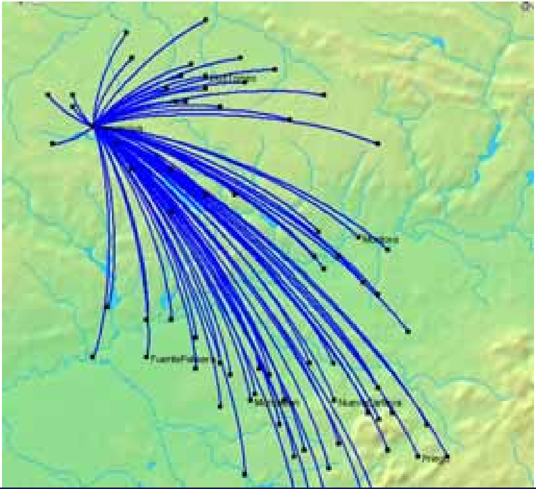
Methodology:

- Solving the problem with objective function 1 (cost)
- Solving the problem with objective function 2 (obnoxious effect)
- Solving a problem
  - Objective function: convex combination of the two objective functions
  - Original constraints
  - Two additional constraints (area to explore)

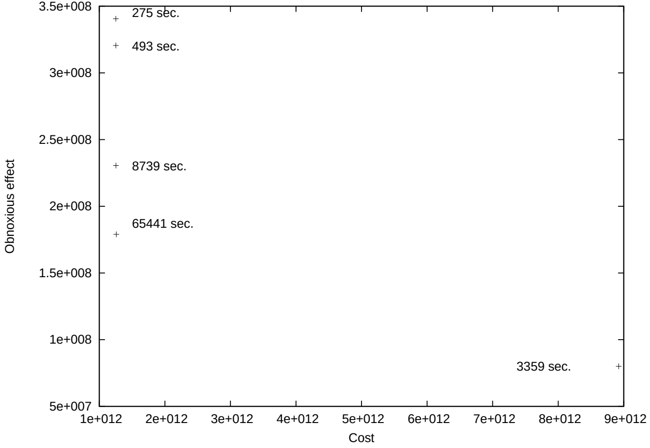
Optimal Solution (Cost). Flows for Scenario 1. USW



Optimal Solution (Obnoxious Effect). Flows for Scenario 1. USW



Non-dominated Solutions



Conclusions and Further Work

- Conclusions
    - Multi echelon, multi commodity, stochastic, bi-objective facility location problem with choice of technology
    - Reasonable computational times
    - Non dominated solutions can be obtained
- Further work
    - More comprehensive models (eg. obnoxious effect dependent on flows/capacities)
    - Efficient solution techniques for large-scale instances

Thank you all

Comments, suggestions, questions are more than welcome